Topology

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Contents

Chapter 1

Topological Spaces

Non-standard examples of topological spaces to check intuition

- (Discrete metric) : $\mathcal{T} = \mathcal{P}(X)$
- (Indiscrete metric) : $\mathcal{T} = \{\phi, X\}$
- $X = \{1, 2, 3\}, \mathcal{T} = \{\phi, \{1\}, \{1, 2\}, \{1, 2, 3\}\}$
- (Co-finite topology) Let X be an infinite set.

$$\mathcal{T} = \{\phi\} \cup \{U \subseteq X : X \setminus U \text{ is finite }\}$$

The indiscrete topology is always the coursest topology, and the discrete topology is always the finest, i.e.

$$\{\phi, X\} \subseteq \mathcal{T} \subseteq \mathcal{P}(X)$$

Remark. We will see many implications of the following form:

TOPOLOGICAL PROPERTY ⇒ PROPERTY IN TERMS OF SEQUENCES

In metric spaces, the converse may sometimes be true. In a general topological setting the converse implication is never true.

CLOSURES We don't think about closures the way we define them. Instead we think about them in terms of the following property:

Proposition 1.

$$\overline{A} = \{x \in X : \text{ for every open } U \subseteq X \text{ with } x \in U, U \cap A \neq \emptyset\}$$

i.e. the set of points such that any open neighborhood of the point intersects A.

From sheet 1 exercise 9: An open set $U \in T$ has non-empty intersection with A if and only if U has non-empty intersection with $\overline{A}$